

1. Answer the following questions.

(a) What is the cardinality of the set $\{\emptyset\}$

(b) What is the power set of $S=\{A,B\}$, where A and B are distinct elements

(c) Is $f(x) = x^2$ a one-to-one function, where $x \in \mathbb{N}$

(d) Is $f(x) = x^2$ an onto function where $x \in \mathbb{R}$?

(e) Is $f(x) = x^3$ an onto function where $x \in \mathbb{R}$?

(f) $\lceil -2.99 \rceil = ?$

(a) 1

(b) $P(S) = \{ \emptyset, \{A\}, \{B\}, \{A,B\} \}$

(c) Y

(d) N

(e) Y

(f) -2

2. Prove that $(A \cup C) \cap [(A \cap B) \cup (B \cap C')] = A \cap B$

$$\begin{aligned} (A \cup C) \cap [(A \cap B) \cup (B \cap C')] &= (A \cup C) \cap [(A \cap B) \cup (C' \cap B)] && \text{(distributive law)} \\ &= (A \cup C) \cap [(A \cup C') \cap B] && \text{(associative law)} \\ &= (A \cup C) \cap (A \cup C') \cap B \\ &= A \cap B \end{aligned}$$

3. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence (denoted as a_n) that begins with the given list. Note that a_n begins with $n=1$. Also, list the next two terms of the sequence based on your derived formula.

(a) 8, 14, 32, 86, 248,

(b) 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682,

(a) $a_n = 3^n + 5$ 734, 2192

(b) $a_{n+1} = 3a_n + 2$ (or $a_n = 3^{n-1} - 1$) 59048, 177146

4. Let $S = \{1,2,3,4\}$ and define functions $f, g: S \rightarrow S$, where $f = \{(1,3), (2,4), (3,1), (4,2)\}$ and $g = \{(1,2), (2,4), (3,3), (4,1)\}$. Find the composite function $g^{-1} \circ f \circ g$.

$$g^{-1} \circ f \circ g = \{(1,2), (2,1), (3,4), (4,3)\}$$